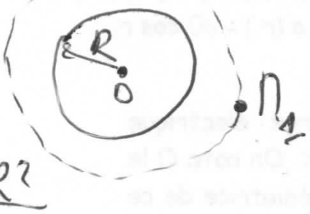
Corrigé type
TD N° 3 — 2011/2012 — 01
Matière : électricité

Exo 1

$$\Phi = \iint_{S_G} \vec{E}\cdot\vec{n}\cdot ds = \frac{\sum q_i}{\varepsilon_0}$$

1. A l'extérieur de la sphère
$r \in [R, +\infty[$

$$E_1 S_G = \frac{Q_{int}}{\varepsilon_0}$$



$$E_1 \cdot 4\pi r^2 = \frac{\sigma \cdot 4\pi R^2}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

$$dV_1 = -E_1\, dr \Rightarrow V_1 = -\int E_1\, dr$$

$$V_1 = -\frac{\sigma R^2}{\varepsilon_0}\int \frac{1}{r^2}\, dr$$

$$V_1 = \frac{\sigma R^2}{\varepsilon_0 r} + C_1$$

$$\left.\begin{array}{l} V_1(\infty) = 0 \\ V_1(\infty) = C_1 \end{array}\right\} \Rightarrow C_1 = 0$$

$$\Rightarrow \boxed{V_1 = \frac{\sigma R^2}{\varepsilon_0 r}}$$

2. A l'intérieur de la sphère M_2
$r \in [0, R[$

$$\Phi = E_2 S_G = \frac{1}{\varepsilon_0} \times 0$$

$$\Rightarrow \boxed{E_2 = 0}$$

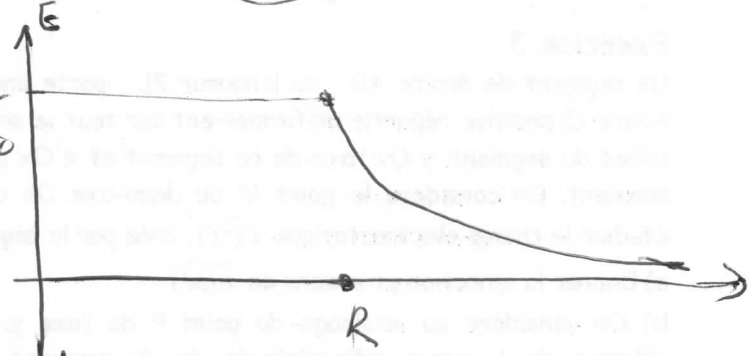
$$V_2 = -\int E_2\, dr$$

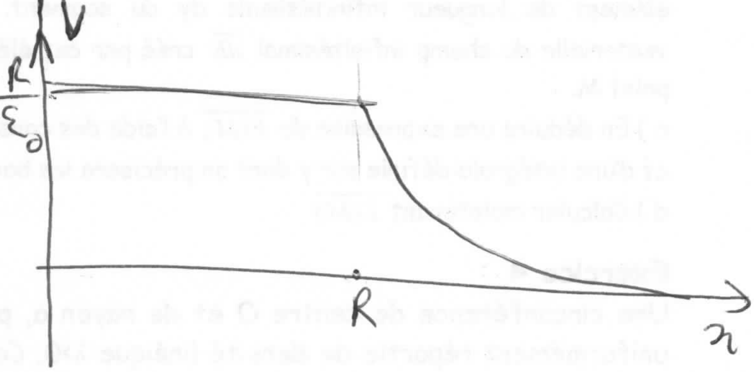
$$V_2 = C_2 = cste$$

$$\lim_{r \to R} V_2 = \lim_{r \to R} V_1$$

$$C_2 = \frac{\sigma R^2}{\varepsilon_0 R} = \boxed{\frac{\sigma R}{\varepsilon_0} = C_2}$$

$$\boxed{V_2 = \frac{\sigma R}{\varepsilon_0}}$$





Exo 2

$$\Phi = \iint_{S_G} \vec{E}\cdot\vec{n}\cdot ds = \frac{\sum Q_i}{\varepsilon_0}$$

1. M_1 est à l'extérieur de cylindre
$r \in [R, +\infty[$

$$\iint_{S_G} \vec{E}\cdot\vec{n}_1\cdot ds + \cancel{\iint_{S_G} \vec{E}\cdot\vec{n}_2\, ds} + \cancel{\iint \vec{E}\cdot\vec{n}_3\cdot ds} = \frac{\sum Q_i}{\varepsilon_0}$$

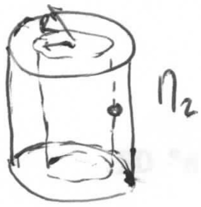
$$E_1 \cdot 2\pi r h = \frac{\rho}{\varepsilon_0}$$

$$\boxed{E_1 = \frac{\rho}{2\pi\varepsilon_0 h\, r}}$$

2. M_2 est à l'intérieur de cylindre

$x \in [0, R[$

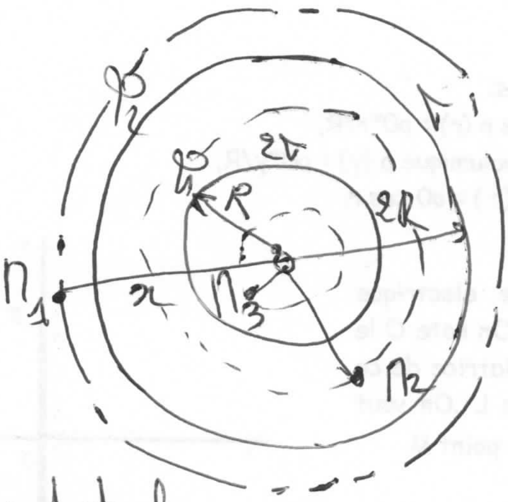
de la même manière



$E_2 \cdot 2\pi x h = \frac{1}{\varepsilon_0} 0$

$E_2 = 0$

Exo 3



1. La charge totale

$Q = Q_1 + Q_2$

$Q_1 = 2\sigma \cdot S_1 = 2\sigma\, 4\pi R^2 = 8\pi\sigma R^2$

$Q_2 = \sigma \cdot S_2 = \sigma\, 4\pi (2R)^2 = 16\pi\sigma R^2$

$Q = 24\pi\sigma R^2$

2. Calcul de champ

$\Phi = \oiint_{SG} \vec{E} \cdot \vec{n}\, ds = \frac{\sum Q_{int}}{\varepsilon_0}$

a. M_1 à l'extérieur $x \in [2R, +\infty[$

$E_1 \cdot 4\pi x^2 = \frac{\sum Q_{int}}{\varepsilon_0} = \frac{24\pi\sigma R^2}{\varepsilon_0}$

$E_1 = \frac{6\sigma R^2}{\varepsilon_0 x^2}$

$V_1 = -\int E_1 dx$

$V_1 = -\frac{6\sigma R^2}{\varepsilon_0} \int \frac{1}{x^2} dx$

$V_1 = \frac{6\sigma R^2}{\varepsilon_0 x} + C_1$

$V_1(\infty) = 0$

$V_1(\infty) = C_1 \Rightarrow C_1 = 0$

$V_1 = \frac{6\sigma R^2}{\varepsilon_0 x}$

2. M_2 entre les deux sphères

$x \in [R, 2R[$

$E_2 \cdot 4\pi x^2 = \frac{8\pi\sigma R^2}{\varepsilon_0}$

$E_2 = \frac{2\sigma R^2}{\varepsilon_0 x^2}$

$V_2 = \frac{2\sigma R^2}{\varepsilon_0 x} + C_2$

$\lim_{x \to 2R} V_1 = \lim_{x \to 2R} V_2$

$\frac{3\sigma R}{\varepsilon_0} = \frac{\sigma R}{\varepsilon_0} + C_2$

$\Rightarrow C_2 = \frac{2\sigma R}{\varepsilon_0}$

$V_2 = \frac{2\sigma R^2}{\varepsilon_0 x} + \frac{2\sigma R}{\varepsilon_0}$

3. M_3 est à l'intérieur

$x \in [0, R[$

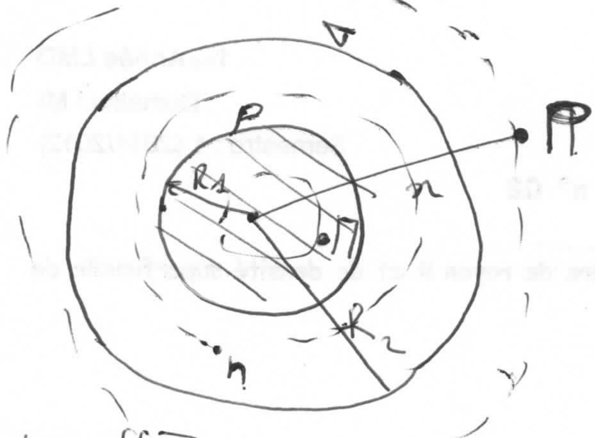
$E_3 \cdot 4\pi x^2 = 0 = E_3$

$E_3 = 0 \Rightarrow V_3 = C_3 = cste$

$\lim_{x \to R} V_3 = \lim_{x \to R} V_2$

$C_3 = \frac{4\sigma R}{\varepsilon} \Rightarrow V_3 = \frac{4\sigma R}{\varepsilon_0}$

Exo 4



$$\Phi = \oiint_{S_G} \vec{E}\cdot\vec{n}\,ds = \frac{\sum q_i}{\varepsilon_0}$$

1. Point P : $r \in [R_2, +\infty[$

$$E_1 \cdot 4\pi r^2 = \frac{\rho \frac{4}{3}\pi R_1^3 + \sigma 4\pi R_2^2}{\varepsilon_0}$$

$$\boxed{E_1 = \frac{\rho R_1^2 (1+R_1)}{3\varepsilon_0 r^2}}$$

2. Point n : $r \in [R_1, R_2[$

$$E_2 \cdot 4\pi r^2 = \frac{\rho \cdot \frac{4}{3}\pi R_1^3}{\varepsilon_0}$$

$$\boxed{E_2 = \frac{\rho R_1^3}{3\varepsilon_0 r^2}}$$

3. Point M : $r \in [0, R[$

$$E_3 \cdot 4\pi r^2 = \frac{\rho \cdot V}{\varepsilon_0}$$

$$E_3 = \frac{\rho \cdot \frac{4}{3}\pi r^3}{4\pi\varepsilon_0 r^2}$$

$$\boxed{E_3 = \frac{\rho r}{3\varepsilon_0}}$$

Exo 5

1. A l'équilibre, les charges se repartissent uniformement sur la surface

2. $\sigma = \frac{Q}{S} = \sigma = \frac{Q}{4\pi R^2}$ (C/m^2)

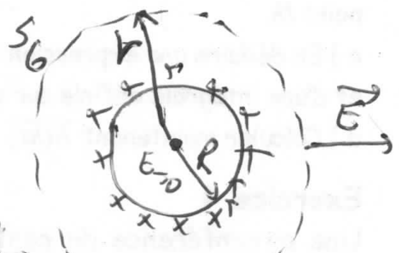
S : surface de la sphère

3. Dans un conducteur à l'équilibre, le champ electrostatique est nul

4. A la surface $E = \frac{\sigma}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

loi de Coulomb

5. pour $r > R$



$$\Phi = E\,S = \frac{Q}{\varepsilon_0}$$

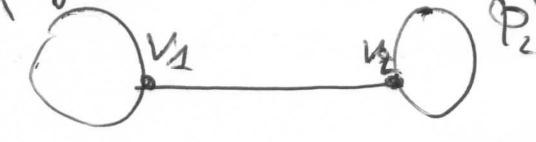
$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\cancel{4\pi} r^2} = \boxed{\frac{Q}{4\pi\varepsilon_0 r^2} = E}$$

Exo 6

Etat initial

Etat final



initiale final

$$q_1 + q_2 = q_1' + q_2'$$ conservation de charge

$$V_1 = V_2$$ continuité de potentiel

$$V_1 = \frac{k q_1'}{R_1}, \quad V_2 = \frac{k q_2'}{R_2}$$

$$\begin{cases} q_1' + q_2' = q_1 \\ \dfrac{q_1'}{R_1} = \dfrac{q_2'}{R_2} \end{cases}$$ système à 2 équations à 2 inconnus

$$q_1' = \frac{R_1}{R_1 + R_2} q_1, \quad q_2' = \frac{R_2}{R_1 + R_2} q_1$$

D. Assas